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Semi-similar solutions of the unsteady compressible second-order boundary layer flow at the stagnation point

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Abstract—Semi-similar solutions of the unsteady compressible laminar boundary layer flow over two-dimensional and axisymmetric bodies at the stagnation point with mass transfer are studied for all the second-order boundary layer effects when the free stream velocity varies arbitrarily with time. The set of partial differential equations governing the unsteady compressible second-order boundary layers representing all the effects are derived for the first time. These partial differential equations are solved numerically using an implicit finite-difference scheme. The results are obtained for two particular unsteady free stream velocity distributions: (a) an accelerating stream and (b) a fluctuating stream. It is observed that the total skin friction and heat transfer are strongly affected by the surface mass transfer and wall temperature. However, their variation with time is significant only for large times. The second-order boundary layer effects are found to be more pronounced in the case of no mass transfer or injection as compared to that for suction.

INTRODUCTION

MANY FLOWS that occur in modern technology possess characteristics that cannot be treated within the single framework of Prandtl's approximation. It is necessary for these more complicated flows, to seek approximate solutions to the Navier–Stokes equations that are of higher accuracy than the classical boundary layer theory. This extension of Prandtl's boundary layer theory is called second-order boundary layer theory which takes into account the effects, which are of the order of magnitude of the boundary layer thickness, i.e. $O(Re^{1/2})$ where Re is the Reynolds number. In general, the second-order effects gain importance when the boundary layer thickness becomes comparable with a characteristic body length. This may be artificially generated by surface mass injection which is of practical interest in the case of transpiration cooling or ablation. Excellent reviews have been given by Van Dyke [1] and Gersten and Gross [2].

Using the method of matched asymptotic expansions first- and second-order boundary layer equations have been obtained from the Navier–Stokes equations [1, 2]. The second-order corrections can be divided into several additive effects [3] such as longitudinal and transverse curvatures, boundary layer displacement, vorticity interaction, velocity slip and temperature jump each of which has a similar physical interpretation.

The steady laminar incompressible and compressible second-order boundary layer theory has been studied by several authors [4–17] for two-dimensional, axisymmetric and three-dimensional stagnation point flow with or without mass transfer. Arunachalam and Rajappa [18] and Afzal and Rizvi [19] have studied unsteady incompressible second-order boundary layer self-similar flow on two-dimensional and axisymmetric bodies at the stagnation point. Recently, the self-similar solution of the analogous compressible case has been obtained in ref. [20] on the assumption that the free stream velocity varies inversely as a linear function of time. Such a distribution of the free stream velocity with time may not be realized in practical situations.

The aim of this investigation is to obtain semi-similar solutions of the unsteady compressible laminar second-order boundary layer flow over two-dimensional and axisymmetric bodies at the stagnation point with mass transfer when the free stream velocity varies arbitrarily with time. The unsteady second-order boundary layers governed by the set of partial differential equations have been obtained for the first time from the Navier–Stokes equations using the method of matched asymptotic expansions. The governing partial differential equations have been solved numerically using an implicit finite-difference scheme. The results have been compared with those of refs. [5, 8, 9] and found to be in good agreement.

GOVERNING EQUATIONS

The unsteady laminar compressible flow of a viscous fluid past two-dimensional and axisymmetric bodies has been considered at the stagnation point. It is assumed that the external flow is homentropic, the surface is maintained at a constant temperature, the dissipation terms are negligible at the stagnation

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NOMENCLATURE

a_1, c_1	accommodation coefficients
$\bar{C}_f$	total skin friction coefficient
C_{f1}	skin friction coefficient due to the first-order boundary layer
f, F	dimensionless first- and second-order stream functions such that $f' = u_1/u_e$, $F' = u_2/u_e$
f_w	mass transfer parameter
$f''(0), F''(0)$	skin friction parameters
g', G	first- and second-order dimensionless temperature, respectively
g'_w	wall temperature
$g''(0), G'(0)$	heat transfer parameters
j	constant which equals 0 for two-dimensional flow and 1 for axisymmetric flow
k	surface curvature of the body
M_∞	free stream Mach number
N_1	stretched variable, y/ε
N	ratio of density viscosity product
p, Pr, Re	pressure, Prandtl number and Reynolds number, respectively
R_{10}, T_{10}	density and temperature at the stagnation point, respectively
q_w	local heat transfer at the wall
$\overline{St}, T$	Stanton number due to both first- and second-order boundary layers and temperature
St_1	Stanton number due to the first-order boundary layer
t_1, t_2	first- and second-order temperature, respectively
t, t^*	dimensional and dimensionless time, respectively
u, v	velocity components in the x- and y-directions, respectively
U_{11}	potential flow velocity gradient in the x-direction
x, y	principal and normal directions, respectively.

Greek symbols

γ	ratio of the specific heats
ε	perturbation parameter, $Re^{-1/2}$
η	similarity variable
μ, ρ	coefficients of viscosity and density, respectively
τ	shear stress at the wall in the x-direction
φ	arbitrary function of time representing unsteadiness in the free stream
ω	exponent in the viscosity law where $\mu \propto T^\omega$.

Superscript

$'$	derivative with respect to η.

Subscripts

d	displacement effect
e, w	conditions at the edge of the boundary layer and on the surface, respectively
L	longitudinal curvature effect
s	velocity slip effect
t, tj	transverse curvature and temperature jump effect, respectively
t^*	derivative with respect to t^*
v	vorticity interaction effect
1, 2	first- and second-order quantities
∞	free stream values.

point, and the free stream velocity varies arbitrarily with time. The first- and second-order boundary layer equations for two-dimensional and axisymmetric flows are obtained from the Navier–Stokes equations using the method of matched asymptotic expansions with a perturbation parameter ε. The outer expansions for u, v, p, ρ, μ and T are of the type

$$u(t,x,y) = U_1(t,x,y) + \varepsilon U_2(t,x,y) + \cdots. \quad (1)$$

The inner expansions for u, $Re^{1/2}\,v$, p, ρ, μ and T are of the type

$$u(t,x,y) = u_1(t,x,N_1) + \varepsilon u_2(t,x,N_1) + \cdots. \quad (2)$$

Substituting the outer and inner expansions (1) and (2) in the Navier–Stokes equations [4–6], we obtain the equation for successive approximations. The matching of inner and outer solutions in the overlap region leads to the first- and second-order boundary layer equations [4–6].

First-order boundary layer equations

$$(Nf'')' + (g' - f')\varphi_{t^*}/\varphi - f'_{t^*} - \varphi f'^2 + \varphi(1+j)ff'' + \varphi g' = 0 \quad (3a)$$

$$(Ng'')' + Pr\,\varphi(1+j)fg'' - Pr\,g'_{t^*} = 0. \quad (3b)$$

The boundary conditions are

$$\left.\begin{array}{ll} f = f_w,\; f' = g = 0,\; g' = g'_w & \text{at } \eta = 0 \\ f' \to 1,\; g' \to 1 & \text{as } \eta \to \infty \end{array}\right\} \text{ for } t^* \geq 0. \quad (4)$$

The initial conditions are given by the steady-state equations obtained by putting $t^* = 0$, $\varphi = 1$, $\partial/\partial t^* = 0$ in equations (3a) and (3b).

Here

$$\eta = (U_{11}/(\rho_{1w}\mu_{1w}))^{1/2}\int_0^\eta \rho_1\,\mathrm{d}N_1,\quad t^* = U_{11}t$$
$$f_w = -(\rho_1 v_1)_w/(\varepsilon U_{11})^{1/2},\quad Re = \rho_\infty U_\infty/(k\mu)$$
$$u_1 = U_{11}x\varphi(t^*)f'(t^*,\eta),\quad t_1 = g'(t^*,\eta)$$
$$v_1 = -(1+j)\rho_1^{-1}(U_{11}\rho_{1w}\mu_{1w})^{1/2}\varphi(t^*)f$$
$$N = \rho_1\mu_1/(\rho_1\mu_1)_w = g'^{\omega-1}. \tag{5}$$

Second-order equations

(1) Longitudinal curvature

$$D_1(F_L,G_L;j) = gf'\varphi_{t^*}/\varphi + gf'_{t^*} - (Nf'')'g + \omega Nf'g'' + \frac{1}{(2+j)}\Big\{2g'(1+j)\varphi\lim_{\eta\to\infty}(g-f) - jf''g'^{\omega} - 2g'(\varphi_{t^*}/\varphi)\lim_{\eta\to\infty}(g-f) + 2g'\lim_{\eta\to\infty}f_{t^*} - 2g'f_{t^*} - j(1+j)\varphi ff'g' + 2\varphi gg' + 2g'(\varphi_{t^*}/\varphi)(g-f)\Big\} - f''\int_0^\eta g_{t^*}\,\mathrm{d}\eta \tag{6a}$$

$$D_2(F_L,G_L;j) = Pr\,gg'_{t^*} - g(Ng'')' - g'^{\omega}g''. \tag{6b}$$

The boundary conditions are

$$\left.\begin{array}{ll} F_L = F'_L = G_L = 0 & \text{at}\ \eta = 0 \\ F'_L \to -g,\quad G_L \to 0 & \text{as}\ \eta\to\infty \end{array}\right\}\ \text{for}\ t^* \geqslant 0. \tag{7}$$

The initial conditions are given by the steady-state equations obtained by putting $t^* = 0$, $\varphi = 1$, $\partial/\partial t^* = 0$ in equations (6a) and (6b). Here

$$u_{2L} = \frac{(U_{11}\rho_{1w}\mu_{1w})^{1/2}}{R_{10}}x\varphi(F'_L + f'G_L)$$
$$v_{2L} = (\rho_{1w}\mu_{1w}/\rho_1 R_{10})\left(\varphi f(1+j)(g-G_L) - (1+j)\varphi F_L - \int_0^\eta (g_{t^*} - G_{Lt^*})\,\mathrm{d}\eta\right)$$
$$t_{2L} = (t_1/R_{10})(\rho_{1w}\mu_{1w}/U_{11})^{1/2}G_L. \tag{8a}$$

The operators D_1 and D_2 are given by

$$D_1(F,G;j) = [N(\omega f''G + F'' + f'G')]' + Nf''G' - (g'-f')(\phi_{t^*}/\phi)G + f'_{t^*}G - \phi(g'G + 2f'F') + (1+j)\phi f(F'' + f'G') + \phi(1+j)f''F - f''\int_0^\eta G_{t^*}\,\mathrm{d}\eta - (\phi_{t^*}/\phi)F' - F'_{t^*} - f'G_{t^*} \tag{8b}$$

$$D_2(F,G;j) = [N(\omega+1)g''G + Ng'G']' + Pr\,\phi(1+j)g''F - Pr\,g'G_{t^*} - Pr\,g''\int_0^\eta G_{t^*}\,\mathrm{d}\eta + Pr\,\phi(1+j)f(g'G' + g''G). \tag{8c}$$

For the case of longitudinal curvature, F and G in equations (8b) and (8c) should be replaced by F_L and G_L.

(2) Transverse curvature

$$[N(\omega f''G_t + f'G'_t + F''_t)]' + Nf''G'_t + 2\phi f''F_t + 2\phi f(F''_t + f'G'_t) - 2\phi f'F'_t - \phi g'G_t - F'_t\phi_{t^*}/\phi + G_t f'_{t^*} - G_t(g'-f')\varphi_{t^*}/\varphi - F'_{tt^*} - f''\int_0^\eta G_{tt^*}\,\mathrm{d}\eta - f'G_{tt^*} =$$
$$g'(Nf')' - g(Nf'')' + Nf'g'' + 2\varphi ff'g' + gf'_{t^*} - 2gg'\varphi_{t^*}\phi + f'g\varphi_{t^*}/\varphi - f'g_{t^*}$$
$$2\varphi gg' - f''\int_0^\eta g_{t^*}\,\mathrm{d}\eta \tag{9a}$$

$$(N(\omega+1)g''G_t + Ng'G'_t)' + 2Pr\,\varphi f(G_t g'' + g'G'_t) + 2Pr\,\varphi g''F_t - Pr\,g'G_{tt^*} - Pr\,g''\int_0^\eta G_{tt^*}\,\mathrm{d}\eta = 2Pr\,\varphi gfg'' - Pr\,g'^{\omega}g'' - Pr\,g''\int_0^\eta g_{t^*}\,\mathrm{d}\eta. \tag{9b}$$

The boundary conditions are

$$\left.\begin{array}{ll} F_t = F'_t = G_t = 0 & \text{at}\ \eta = 0 \\ F'_t \to g,\quad G_t \to 0 & \text{as}\ \eta\to\infty \end{array}\right\}\ \text{for}\ t^* \geqslant 0 \tag{10}$$

where

$$u_{2t} = (U_{11}\rho_{1w}\mu_{1w})^{1/2}R_{10}^{-1}x\varphi(F'_t + f'(G_t - g))$$
$$v_{2t} = (\rho_{1w}\mu_{1w}/\rho_1 R_{10}) \times \left(2\varphi(fg - fG_t - F_t) - \int_0^\eta (g_{t^*} - G_{tt^*})\,\mathrm{d}\eta\right)$$
$$t_{2t} = (t_1/R_{10})(\rho_{1w}\mu_{1w}/U_{11})^{1/2}G_t. \tag{11}$$

As mentioned earlier the initial conditions are given by the steady-state equations corresponding to equations (9a) and (9b).

(3) Boundary layer displacement

$$D_1(F_d,G_d;j) = -g'(\varphi_{t^*} + 2\varphi^2)/\varphi \tag{12a}$$
$$D_2(F_d,G_d;j) = 0. \tag{12b}$$

The boundary conditions are

$$\left.\begin{array}{ll} F_d = F'_d = G_d = 0 & \text{at}\ \eta = 0 \\ F'_d \to 1,\quad G_d \to 0 & \text{as}\ \eta\to\infty \end{array}\right\}\ \text{for}\ t^* \geqslant 0 \tag{13}$$

where

$$u_{2d} = x\varphi(F'_d + f'G_d)$$

$$v_{2d} = -\rho_1^{-1}(\rho_{1w}\mu_{1w}/U_{11})^{1/2} \times\left((1+j)\varphi(F_d+fG_d) - \int_0^\eta G_{dt^*}\,d\eta\right)$$

$$t_{2d} = (t_1/U_{11})G_d. \tag{14}$$

(4) Vorticity interaction

$$D_1(F_v, G_v; j) = (1+j)\lim_{\eta\to\infty}(g-f) \tag{15a}$$

$$D_2(F_v, G_v; j) = 0 \tag{15b}$$

$$\left.\begin{array}{ll} F_v = F'_v = G_v = 0 & \text{at } \eta = 0 \\ F'_v \to -g, \quad G_v \to 0 & \text{as } \eta \to \infty \end{array}\right\} \text{ for } t^* \geqslant 0 \tag{16}$$

where

$$u_{2v} = R_{10}^{-1}(\rho_{1w}\mu_{1w}/U_{11})^{1/2}x\varphi(F'_v+f'G_v)$$

$$v_{2v} = -(\rho_{1w}\mu_{1w}/\rho_1 R_{10}U_{11}) \times\left((1+j)\varphi(fG_v+F_v) - \int_0^\eta G_{vt^*}\,d\eta\right)$$

$$t_{2v} = (\rho_{1w}\mu_{1w}/U_{11})^{1/2}(t_1/U_{11}R_{10})G_v. \tag{17}$$

(5) Velocity slip

$$D_1(F_s, G_s; j) = 0 \tag{18a}$$

$$D_2(F_s, G_s; j) = 0. \tag{18b}$$

The boundary conditions are

$$\left.\begin{array}{lll} F_s = G_s = 0, & F'_s = f''(0) & \text{at } \eta = 0 \\ F'_s = G_s = 0 & & \text{as } \eta \to \infty \end{array}\right\} \text{ for } t^* \geqslant 0 \tag{19}$$

where

$$u_{2s} = \gamma^{1/2}M_\infty t_w^{\omega-1/2}(U_{11}/\rho_{1w}\mu_{1w})^{1/2}U_{11}x\varphi(F'_s+f'G_s)$$

$$v_{2s} = -U_{11}\rho_1^{-1}\gamma^{1/2}M_\infty t_w^{\omega-1/2} \times\left(\phi(1+j)(F_s+fG_s) - \int_0^\eta G_{st^*}\,d\eta\right)$$

$$t_{2s} = \gamma^{1/2}M_\infty t_w^{\omega-1/2}(U_{11}/\rho_{1w}\mu_{1w})^{1/2}t_1 G_s. \tag{20}$$

(6) Temperature jump

$$D_1(F_{tj}, G_{tj}; j) = 0 \tag{21}$$

$$D_2(F_{tj}, G_{tj}; j) = 0. \tag{22}$$

The boundary conditions are

$$\left.\begin{array}{lll} F_{tj} = F_{tj} = 0, & G_{tj} = g''(0)/g'(0) & \text{at } \eta = 0 \\ F'_{tj} = G_{tj} = 0 & & \text{as } \eta \to \infty \end{array}\right\} \text{ for } t^* \geqslant 0 \tag{23}$$

where u_{2tj}, v_{2tj} and t_{2tj} are given by equations (20) by replacing s by tj. The operators D_1 and D_2 which occur in equations (12)–(22) are defined in equations (8b) and (8c) where F and G have to be replaced by F_d and G_d for boundary layer displacement by F_v and G_v for vorticity interaction, by F_s and G_s for velocity slip, and by F_{tj} and G_{tj} for temperature jump.

It may be remarked that the initial conditions in the case of boundary layer displacement, vorticity interaction, velocity slip and temperature jump are given by the corresponding steady-state equations. The steady-state equations are obtained by putting $t^* = 0$, $\phi = 1$, $\partial/\partial t^* = 0$ in equations (12), (15), (18), (21) and (22).

The skin friction and heat transfer coefficients are given as follows.

The first-order skin friction and heat transfer coefficients are given by

$$C_{f1} = \tau_{w1}/(U_{11}^3 R_{10} T_{10}^\omega)^{1/2} = g_w'^{(\omega-1)/2}x\varphi(t^*)f''(0) \tag{24a}$$

$$St_1 = -q_{w1}\,Pr/(U_{11}R_{10}T_{10}^\omega)^{1/2} = g_w'^{(\omega-1)/2}2g''(0). \tag{24b}$$

The second-order skin friction and heat transfer coefficients can be written as follows.

For longitudinal curvature the skin friction and heat transfer coefficients (C_{fL}, St_L) are given by

$$C_{fL} = \tau_{wL}/(U_{11}T_{10}^\omega) = g_w'^{\omega-1}x\varphi(t^*)F''_L(0) \tag{25a}$$

$$St_L = -q_{wL}\,Pr/T_{10}^{\omega+1} = g_w'^{\omega}G'_L(0). \tag{25b}$$

The skin friction and heat transfer coefficients for the transverse curvature effect (C_{ft}, St_t) can be obtained by changing the subscript L by t in the set of equations (25).

For the boundary layer displacement effect

$$C_{fd} = \tau_{wd}/(U_{11}R_{10}T_{10}^\omega)^{1/2} = xg_w'^{(\omega-1)/2}\varphi(t^*)F''_d(0) \tag{26a}$$

$$St_d = -q_{wd}\,Pr\,U_{11}^{1/2}/(R_{10}T_{10}^{\omega+2})^{1/2} = g_w'^{(\omega+1)/2}G'_d(0). \tag{26b}$$

For the vorticity interaction effect

$$C_{fv} = \tau_{wv}/T_{10}^\omega = xg_w'^{\omega-1}\varphi(t^*)F''_v(0) \tag{27a}$$

$$St_v = -q_{wv}\,Pr\,U_{11}/T_{10}^{\omega+1} = g_w'^{\omega}G'_v(0). \tag{27b}$$

For the velocity slip effect

$$C_{fs} = \tau_{ws}/(\gamma^{1/2}M_\infty U_{11}^2 T_{10}^{\omega-1/2}) = xg_w'^{(\omega-1/2)}\varphi(t^*)F''_s(0) \tag{28a}$$

$$St_s = -q_{ws}\,Pr/(\gamma^{1/2}M_\infty U_{11}T_{10}^{\omega+1/2}) = g_w'^{\omega+1/2}G'_s(0). \tag{28b}$$

For the temperature jump effect

$$C_{ftj} = \tau_{wtj}/(\gamma^{1/2}M_\infty U_{11}^2 T_{10}^{\omega-1/2}) = xg_w'^{\omega-1/2}\varphi(t^*)[F''_{tj}(0) + (1+\omega)f''(0)G_{tj}(0)] \tag{29a}$$

$$St_{tj} = -q_{wtj}\,Pr/(\gamma^{1/2}M_\infty U_{11}T_{10}^{\omega+1/2}) = g_w'^{\omega+1/2}[G'_{tj}(0) + (1+\omega)g''(0)G_{tj}(0)]. \tag{29b}$$

The total skin friction $\bar{C}_f$ (i.e. first order plus second order) and the total heat transfer $\overline{St}$ (i.e. first order plus second order) are given by

$$\bar{C}_f = C_{f1} + \varepsilon(C_{fL} + kC_{ft} + U_{21}C_{fd} + \Omega C_{fv} + a_1 C_{fs} + c_1 C_{ftj}) \tag{30a}$$

$$\overline{St} = St_1 + \varepsilon(St_L + k\,St_t + U_{21}\,St_d + \Omega\,St_v + a_1\,St_s + c_1\,St_{tj}) \quad (30b)$$

$$C_f = \bar{C}_f / x g_w'^{(\omega-1)/2} \varphi \quad (31a)$$

$$St = \overline{St} / g_w'^{(\omega-1)/2}. \quad (31b)$$

RESULTS AND DISCUSSION

The non-linear first-order boundary layer equation (3) with boundary condition (4) has been solved using the quasilinearization technique in combination with an implicit finite-difference scheme. Then the linear second-order boundary layer equations (6), (9), (12), (15), (18), (21) with boundary conditions (7), (10), (13), (16), (19), (22) have been solved using an implicit finite-difference scheme. Since the method has been described in detail in refs. [21–23] it is not reported here for the sake of brevity. The step sizes $\Delta\eta$ and Δt^* are optimized and $\Delta\eta = 0.05$ and $\Delta t^* = 0.1$ are used throughout the computation. The boundary layer edge (η_∞) is also optimized and it is found that it depends on the injection parameter (f_w) and wall temperature (g_w'). It is assumed that U_{21} is negative [7, 15].

In order to assess the accuracy of our method we have compared our second-order boundary layer heat transfer results due to curvature, displacement and vorticity (St_c, St_d, St_v) for the two-dimensional steady flow with those of Fannelöp and Flügge-Lötz [9], the axisymmetric results with those of Van Dyke [5] and Davis and Flügge-Lötz [8] and found them to be in good agreement. The comparison is shown in Fig. 1.

Computations have been carried out for various values of the mass transfer parameter (f_w), wall temperature (g_w') and for two different unsteady free stream velocity distributions which have a continuous first derivative for all t^* characterized by $\varphi(t^*) = 1 + \varepsilon_1 t^{*2}$ and $\varphi(t^*) = [1 + \varepsilon_2 \cos(\omega^* t^*)]/(1 + \varepsilon_2)$ where $\varepsilon_1 = 0.25$, $\varepsilon_2 = 0.1$ and $\omega^* = 5.6$. Results for the accelerated stream and fluctuating flow for two-dimensional flow are shown in Figs. 2–4 and for axisymmetric flow in Fig. 5.

The effect of mass transfer (f_w) on the total skin friction and heat transfer (C_f, St), which include the effects of both first- and second-order boundary layers, is shown in Fig. 2. The skin friction and heat transfer due to the first-order boundary layer only (C_{f1}, St_1) are also shown in Fig. 2. It is found that the total skin friction and heat transfer (C_f, St) differ significantly when $f_w \leqslant 0$ (injection and no mass transfer) and this difference is less in the case of suction ($f_w > 0$), because suction ($f_w > 0$) reduces both the first- and second-order boundary layer thicknesses. The effect of injection ($f_w < 0$) is just the opposite. For example, for $t^* = f_w = 1$, the difference between

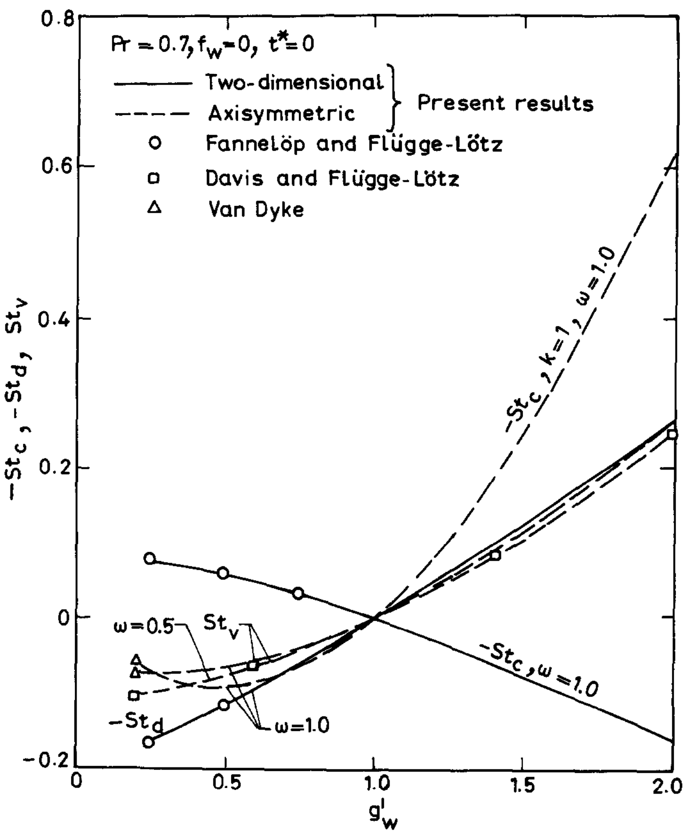

FIG. 1. Comparison of second-order boundary layer heat transfer coefficients due to curvature, displacement and vorticity (St_c, St_d, St_v) for the steady flow.

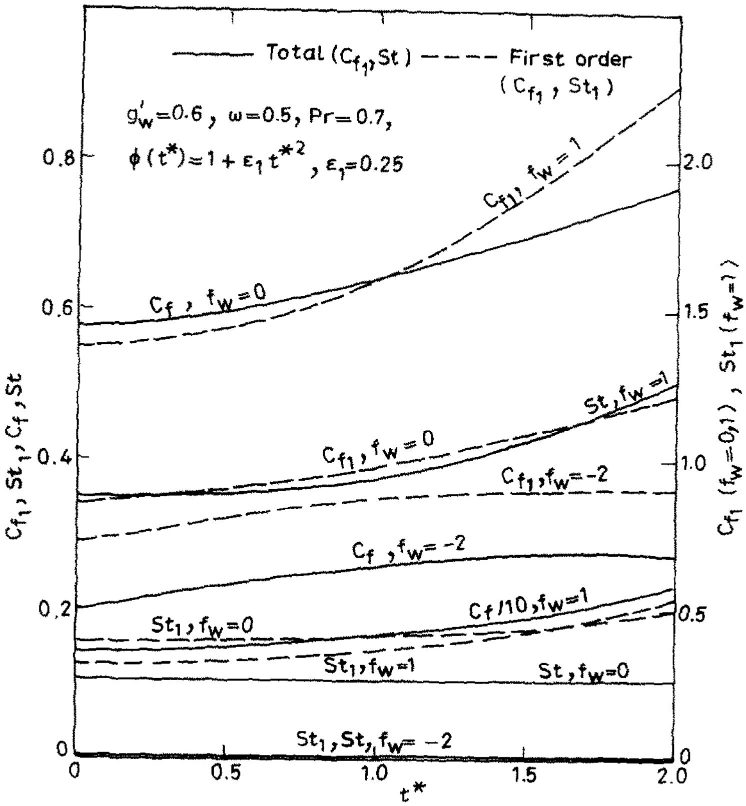

Fig. 2. Effect of mass transfer (f_w) on the total skin friction and heat transfer (C_f, St) and the skin friction and heat transfer due to the first-order boundary layer (C_{f1}, St_1) for the two-dimensional flow ($j = 0$) when $\phi(t^*) = 1+\varepsilon_1 t^{*2}$.

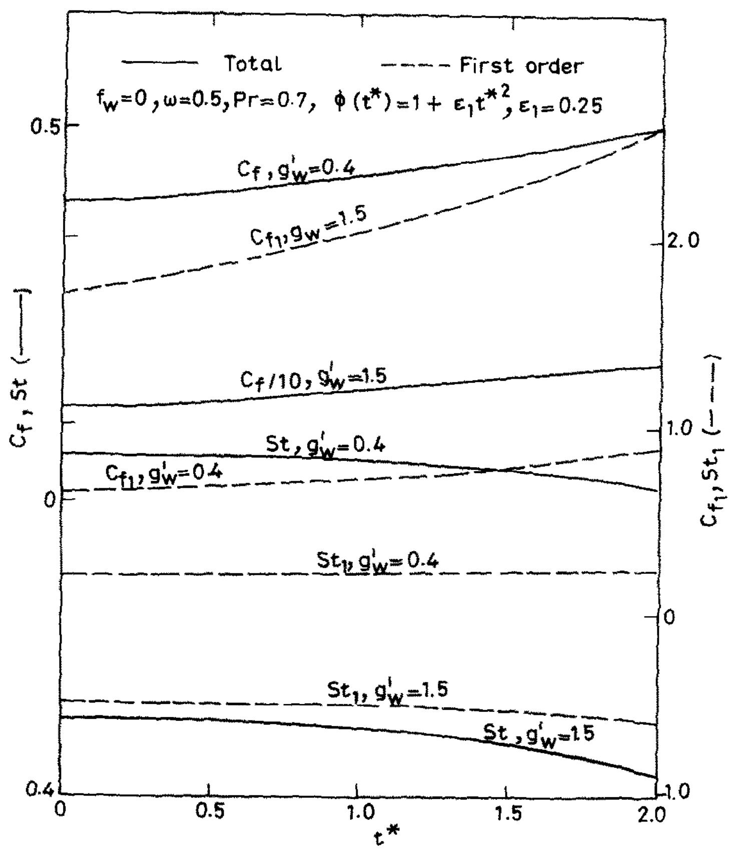

Fig. 3. Effect of wall temperature (g'_w) on the total skin friction and heat transfer (C_f, St) and the skin friction and heat transfer due to the first-order boundary layer (C_{f1}, St_1) for the two-dimensional flow ($j = 0$) when $\phi(t^*) = 1+\varepsilon_1 t^{*2}$.

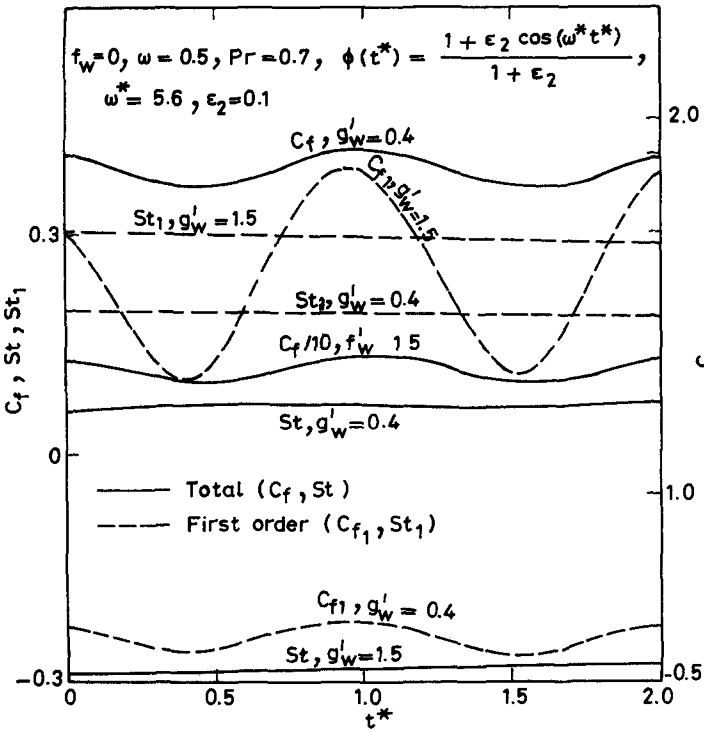

FIG. 4. Effect of wall temperature (g'_w) on the total skin friction and heat transfer (C_f, St) and the skin friction and heat transfer due to the first-order boundary layer (C_{f1}, St_1) for the two-dimensional flow ($j = 0$) when $\phi(t^*) = [1+\varepsilon_2 \cos(\omega^* t^*)]/(1+\varepsilon_2)$.

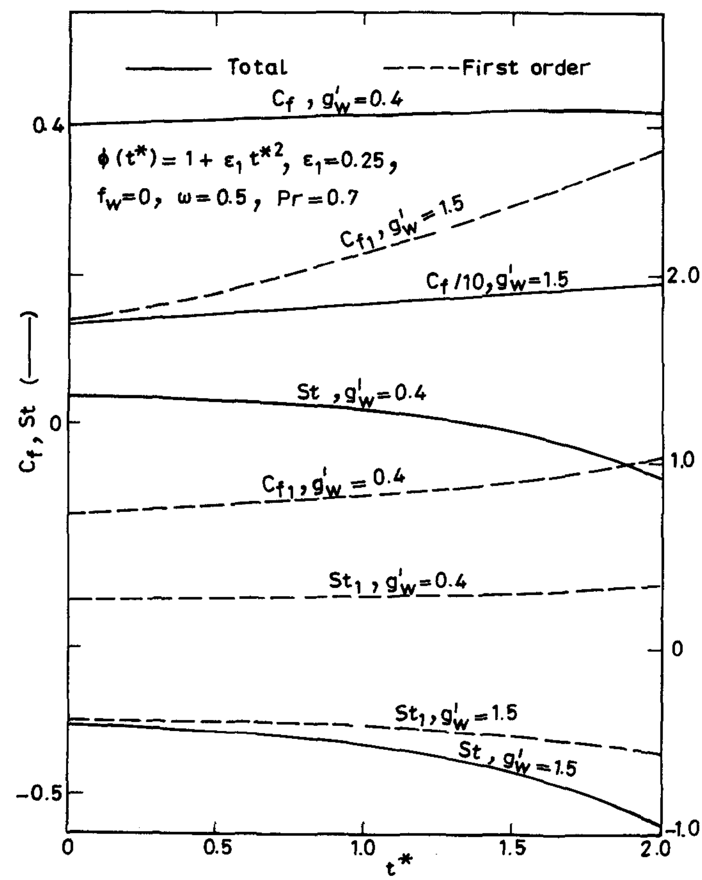

FIG. 5. Effect of wall temperature (g'_w) on the total skin friction and heat transfer (C_f, St) and the skin friction and heat transfer due to the first-order boundary layer (C_{f1}, St_1) for the axisymmetric flow ($j = 1$) when $\phi(t^*) = 1+\varepsilon_1 t^{*2}$.

C_f and C_{f1} is about 3% and the difference between St and St_1 is about 7%. On the other hand, for $f_w \leqslant 0$ the difference between C_f and C_{f1} is more than 30%. The heat transfer St is about one-quarter of that of the first order. For $f_w = -2$, both St and St_1 are very small. Therefore, the second-order boundary layer effects become very important in the case of injection ($f_w < 0$) and no mass transfer ($f_w = 0$) as compared to the case of suction ($f_w > 0$). It is also observed that the total skin friction and heat transfer (C_f, St) for $f_w \leqslant 0$ are less than those of the first-order boundary layers (C_{f1}, St_1). However, for $f_w > 0$ they are slightly more than that of the first-order boundary layers.

Both skin friction and heat transfer (C_f, St) are strongly affected by the mass transfer parameter (f_w) and they decrease due to injection ($f_w < 0$) and increase due to suction ($f_w > 0$). This behaviour is due to the increase in the boundary layer thickness due to injection ($f_w < 0$) and the effect of suction ($f_w > 0$) is just the reverse. For accelerating flow ($\varphi(t^*) = 1+\varepsilon_1 t^{*2}$) the total skin friction and heat transfer (C_f, St) increase with time t^*. However, the effect is significant only for large time t^* ($t^* \geqslant 1$). Also, for $f_w \leqslant 0$, the change in heat transfer (St) with t^* is very small. It may be remarked that for decelerating flow ($\varphi(t^*) = 1-\varepsilon_1 t^{*2}$), both skin friction and heat transfer (C_f, St) decrease as t^* increases. The results are not shown here for the sake of brevity.

The effect of wall temperature (g'_w) on the skin friction and heat transfer due to both first- and second-order boundary layers (C_f, St) is shown in Fig. 3. The skin friction and heat transfer due to first-order boundary layers (C_{f1}, St_1) are also shown in Fig. 3. Both skin friction and heat transfer (C_f, St) are strongly dependent on the wall temperature (g'_w). For the hot wall ($g'_w > 1$), $St < 0$. This implies that the heat is transferred from the wall to the fluid. For the cold wall ($g'_w < 1$), $St > 0$ which implies that there is a transfer of heat from the fluid to the wall. The skin friction and heat transfer change significantly only for large time t^*. It is seen that for both cold wall ($g'_w = 0.4$) and hot wall ($g'_w = 1.5$), total skin friction and heat transfer (C_f, St) are less than those of the first order (C_{f1}, St_1) when $f_w = 0$.

The effect of wall temperature (g'_w) on the total skin friction and heat transfer (C_f, St) for the fluctuating free stream velocity $\varphi(t^*) = [1+\varepsilon_2 \cos(\omega^* t^*)]/(1+\varepsilon_2)$ is presented in Fig. 4. Figure 4 also contains the corresponding results (C_{f1}, St_1) of the first-order boundary layers. Since the first-order boundary layer results are discussed in detail in ref. [24], they are not discussed here. It is observed that the total skin friction (C_f) responds more to the fluctuations of the free stream than the total heat transfer (St). In fact, the response of the total heat transfer (St) is very small. The reason for such a behaviour is that the skin friction is directly proportional to the velocity gradient which is strongly affected by the fluctuations in the free stream. On the other hand, the heat transfer is directly proportional to the temperature gradient which is somewhat indirectly affected by the fluctuations of the free stream velocity. Like in accelerating flow, the total skin friction and heat transfer (C_f, St) are strongly affected by the wall temperature g'_w.

The effect of the wall temperature (g'_w) on the total skin friction and heat transfer (C_f, St) and on the boundary layer results (C_{f1}, St_1) for the axisymmetric flow ($j = 1$) is shown in Fig. 5. Since the effect of g'_w on C_f and St is qualitatively similar to that of the two-dimensional flow ($j = 0$) given in Fig. 5, the results are not discussed here.

Since we have presented results taking into account the first-order boundary layer effects and all the second-order boundary layer effects, for the sake of brevity, the results showing the curvature effects, displacement effects, vorticity effects, and velocity slip and temperature jump effects are not presented here separately.

CONCLUSIONS

The governing partial differential equations for unsteady compressible second-order boundary layers describing all the effects in the case of a two-dimensional and axisymmetric stagnation point flow have been derived for the first time. The results indicate that the total skin friction and heat transfer for both two-dimensional and axisymmetric cases are strongly dependent on the surface mass transfer and wall temperature. Both the total skin friction and heat transfer decrease due to injection and the effect of suction is just the reverse. The total skin friction and heat transfer for the accelerating flow change significantly only for large times. The skin friction responds more to the fluctuations of the free stream than the heat transfer. The second-order boundary layer effects are found to be more significant in the presence of injection and no mass transfer as compared to that of suction.

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SOLUTIONS SEMI-SIMILAIRES D'UN ECOULEMENT VARIABLE COMPRESSIBLE DE COUCHE LIMITE DE SECOND ORDRE, PRES DU POINT D'ARRET

Résumé—Des solutions semi-similaires d'écoulement variable compressible de couche limite sur des corps bi-dimensionnels thermique, sont étudiées pour tous les effets de couche limite du second ordre, lorsque la vitesse de l'écoulement libre varie arbitrairement avec le temps. Le système d'équations aux dérivées partielles représentant tous les effets est écrit pour la première fois. On le résout numériquement à l'aide d'un schéma implicite aux différences finies. Les résultats sont obtenus pour deux cas de vitesse variable d'écoulement libre : (a) un écoulement accéléré et (b) un écoulement fluctuant. On observe que le frottement pariétal total et le transfert de chaleur sont fortement affectés par le transfert de masse et la température pariétaux. Néanmoins, leur variation avec le temps est sensible seulement pour des grandes durées. Les effets sont trouvés plus prononcés dans le cas de l'absence du transfert de masse ou de l'injection par rapport au cas de l'aspiration.

QUASI-SIMULTANE LÖSUNG EINER INSTATIONÄREN KOMPRESSIBLEN GRENZSCHICHTSTRÖMUNG ZWEITER ORDNUNG AM STAGNATIONSPUNKT

Zusammenfassung—Unter Berücksichtigung des Massentransports werden quasi-simultane Lösungen von instationären Grenzschichtströmungen am Stagnationspunkt an zwei-dimensionalen, achsensymmetrischen Körpern untersucht. Dabei werden alle Grenzschichteffekte zweiter Ordnung aufgezeigt, die sich bei zeitlich beliebig veränderlicher freier Anströmgeschwindigkeit ergeben. Zum ersten Mal wird das gesamte System von partiellen Differentialgleichungen hergeleitet, welches alle Effekte der instationären kompressiblen Grenzschicht zweiter Ordnung beschreiben kann. Die partiellen Differentialgleichungen werden numerisch mit Hilfe eines impliziten Differenzenverfahrens gelöst. Die Ergebnisse für zwei spezielle instationäre Geschwindigkeitsverteilungen, (a) einer beschleunigten Strömung und (b) einer schwankenden Strömung, werden ermittelt. Es wird dabei beobachtet, daß die Gesamtreibung an der Oberfläche und der Wärmeübergang sehr stark vom Massentransport an der Oberfläche und von der Wandtemperatur abhängen. Trotzdem ist deren zeitliche Änderung nur für große Zeiten signifikant. Es zeigt sich, daß im Fall von verschwindendem Massentransport oder Einblasung die Grenzschichteffekte zweiter Ordnung im Gegensatz zur Absaugung stärker in Erscheinung treten.

ПОЛУАВТОМОДЕЛЬНЫЕ РЕШЕНИЯ ДЛЯ НЕСТАЦИОНАРНОГО СЖИМАЕМОГО ТЕЧЕНИЯ В ПОГРАНИЧНОМ СЛОЕ 2-ГО ПОРЯДКА В ТОЧКЕ ТОРМОЖЕНИЯ

Аннотация—При произвольном изменении скорости потока во времени изучаются полуавтомодельные решения для всех эффектов второго порядка нестационарного течения сжимаемого газа с учетом массообмена в пограничном слое вблизи точки торможения двумерных осесимметричных тел. Впервые выведена система дифференциальных уравнений в участных производных, определяющих нестационарные сжимаемые пограничные слои второго порядка с учетом всех их эффектов. Эти уравнения решены численно на основе неявной конечно-разностной схемы. Получены результаты для двух конкретных случаев распределния скоростей в нестационарном набегающем потоке: (а) при течении с ускорением и (б) при колебаниях скорости течения. Замечено, что суммарное поверхностное трение и теплоперенос сильно зависят от массопереноса на поверхности и температуры стенок. Однако, они претерпевают существенное изменение только на больших отрезках времени. Найдено, что эффекты пограничного слоя второго порядка наиболее сильно выражены при отсутствии массопереноса или вдува по сравнению со случаем отсоса пограничного слоя.